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Programming the Procurement of Airlift and Sealift Forces: A Linear Programming Model for Analysis of the Least-Cost Mix of Strategic Deployment Systems

RAC

LOGISTICS DEPARTMENT
RAC PAPER RAC-P-17
Published March 1966
Second Printing May 1966
Revised September 1966

Programming the Procurement of Airlift and Sealift Forces: A Linear Programming Model for Analysis of the Least-Cost Mix of Strategic Deployment Systems

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FOREWORD

This paper is printed as a courtesy to the authors. It was presented at the 1966 American Meeting of The Institute of Management Science, Dallas, Tex., 17 February 1966.

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ABSTRACT

A linear programming model for analyzing the strategic deployment mix of airlift and sealift forces and prepositioning to accomplish the composite requirements of a complex of possible contingencies is described in this paper. It solves for the least-cost mix of deployment means capable of meeting any one of a spectrum of contingencies, or meeting simultaneous contingencies. The model was developed by RAC as part of the US Army's study program and has been used in analyses of deployment systems conducted in support of the Joint Chiefs of Staff and the Office of the Secretary of Defense. Results of analyses have influenced the preparation of long-range plans as well as the formulation of the FY67 Department of Defense budget. The paper gives the background and assumptions of the model, describes the model by means of a simple hypothetical example followed by a selected subset of a complete version, and discusses how the model is used.

INTRODUCTION

To further national objectives the US, through treaties and other means, has incurred military commitments around the world. To meet these commitments should they fall due, limited active military forces are maintained. Pending the call-up and deployment of additional forces, should such action prove necessary, means must be provided to move active forces quickly to wherever they may be required—almost anywhere on the globe. Possible means for providing this kind of strategic mobility include prepositioning of forces abroad, stockpiling of materiel overseas, creation of fleets of transport aircraft, creation of fleets of fast ships, and combinations of these and other alternatives. How best to achieve the necessary level of strategic mobility is and will continue to be an important problem facing military planners.

The primary purpose of this paper is to describe a deployment model developed at RAC for the analysis of strategic mobility problems and to indicate in some detail the various capabilities of this model. The chief use of the model has been in performing cost-effectiveness evaluations of strategic deployment systems. The Office of the Assistant Secretary of Defense (Systems Analysis) [OASD(SA)] worked closely with the RAC study group in defining measures of effectiveness and problems to be addressed by the model, in providing cost inputs and other key input data, and in describing those sensitivity analyses to be performed with the model that would be particularly useful in resolving some of the major uncertainties in deployment system evaluations. Hence the model was sufficiently responsive to Department of Defense needs to serve as a key input to 1965 force structure evaluations.

Previous studies of strategic mobility have generally focused on typical deployments to one part of the world or another. In some instances the so-called "worst case" has been postulated on the premise that the capability to meet that requirement embraces all others. The model described in this paper is capable of analyzing simultaneously the composite requirements of a set of deployment requirements located throughout the world. The requirements may be imposed singly or in simultaneous combinations. The outputs of the linear programming model to be described are combinations of deployment means from among those available, programmed in such a manner that specified requirements can be met at minimum cost. Because of the interrelations among requirements, the capabilities of the means being analyzed, and costs, changes in the values of any of these factors will have an impact on the others. The model provides an efficient means for analyzing these interrelations.

DESCRIPTION OF THE MODEL USING A SIMPLE HYPOTHETICAL EXAMPLE

The type of problem the model is designed to solve can be illustrated simply. Consider a country A that has defense agreements with two other

countries, B and C, which are distant from A but relatively close to each other. B is closer to A than is C. The geographical relation of these countries to each other is portrayed schematically in Fig. 1.

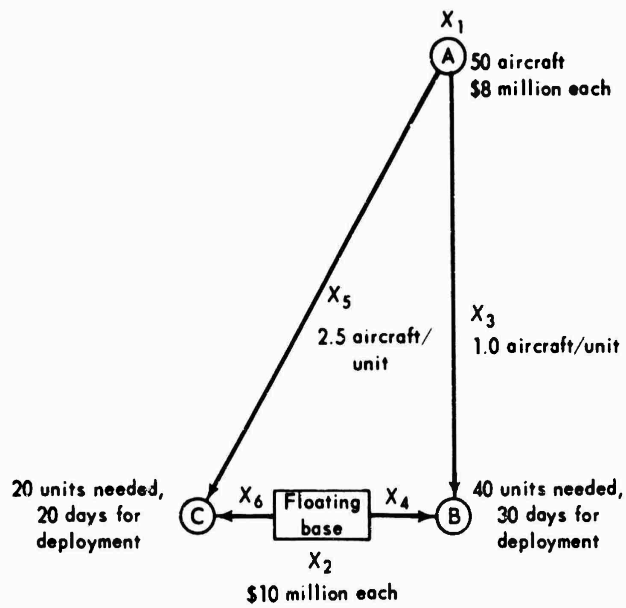

Fig. 1—Simplified Example of a Multitheater Strategic Deployment Problem

X_1 = transport aircraft retained in the fleet; X_2 = force units prestocked in floating-depot ships; X_3 = force units delivered to B by aircraft; X_4 = force units delivered to B by floating-depot ships; X_5 = force units delivered to C by aircraft; and X_6 = force units delivered to C by floating-depot ships.

Military planners of A have determined that B can be effectively supported if a force of 40 units can be deployed within a period of 30 days. A contingency in C, on the other hand, would require a force of only 20 units, but only 20 days may be allowed for its deployment. Country A has in its inventory a fleet of 50 transport aircraft, and no more may be secured. The investment cost of these aircraft is considered sunk by A, but the 10-year operating cost of each aircraft retained in inventory will be $8 million. Country A also has the option to buy floating-depot ships and military equipment at a cost, including initial investment and 10 years of operation, of $10 million per force unit prestocked in the ships. Country A desires a deployment system capable of meeting the requirements of either B or C. The only components available for such a deployment system are the fleet of aircraft in inventory and the floating bases that may be procured. The problem is to find the least-cost combination of the available means possessing the desired capability.

Formulation of the problem requires the use of six variables:

X_1 = transport aircraft retained in the fleet
X_2 = force units prestocked in floating-depot ships
X_3 = force units delivered to B by aircraft
X_4 = force units delivered to B by floating-depot ships
X_5 = force units delivered to C by aircraft
X_6 = force units delivered to C by floating-depot ships

Using these variables a linear programming problem may be formulated as follows: Subject to the following constraints,

$$X_1 \leq 50 \tag{1}$$

$$X_3 + X_4 = 40 \tag{2}$$

$$X_1 - 1.0\,X_3 \geq 0 \tag{3}$$

$$X_2 - X_4 \geq 0 \tag{4}$$

$$X_5 + X_6 = 20 \tag{5}$$

$$X_1 - 2.5\,X_5 \geq 0 \tag{6}$$

$$X_2 - X_6 \geq 0 \tag{7}$$

choose $X_j \geq 0$, $j = 1, \ldots, 6$ to minimize the 10-year cost, and the cost function is

$$8X_1 + 10X_2.$$

Constraint 1 states that the number of aircraft retained in inventory cannot exceed the 50 that are on hand.

Constraint 2 states that the sum of the force units delivered by aircraft and by floating-depot ships to B must equal the 40 force units required. Constraint 3 states that the number of aircraft deploying forces to B cannot exceed the number of aircraft retained in the fleet. The coefficient of X_3 in constraint 3 is unity. This coefficient, which is in units of aircraft per force unit, indicates the aircraft required to lift a force unit to B and is a function of the aircraft productivity over the route from A to B and the time allowed for the deployment. Constraint 4 states that the number of force units delivered to B by floating depot cannot exceed the number of units stockpiled in the floating base.

Constraints 5 to 7 are similar to constraints 2 to 4 except that they relate to a deployment to C. Note that in constraint 6, however, the coefficient for units of aircraft per force unit, used to compute the aircraft required to lift X_5 force units to C, is 2.5 aircraft per force unit. Because the distance from A to C is greater than from A to B and the deployment period is shorter, 2.5 aircraft are required to deliver one force unit to C.

Finally, the cost function expresses the 10-year cost of the deployment system for any values of X_1 and X_2, the variables specifying the components of the system. It is this function that is to be optimized (minimized) while satisfying constraints 1 to 7. Any set of nonnegative values for the variables

X_1 to X_6 that minimizes the cost function while satisfying constraints 1 to 7 constitutes an optimal solution to the problem.

The optimal solution with values of the variables rounded to the nearest integer is $X_1 = 33$, $X_2 = 7$, $X_3 = 33$, $X_4 = 7$, $X_5 = 13$, and $X_6 = 7$.

The least-cost deployment system, as indicated by the values of X_1 and X_2, consists of a fleet of 33 aircraft and sufficient floating-depot ships to prestock 7 force units. Substitution of these values in the cost function gives a 10-year system cost of $334 million. The solution indicates that it would be economical to retire 17 aircraft from the original fleet and that the remaining 33 aircraft can deploy 33 force units to B or 13 force units to C. The balance of the requirement of B or C can be delivered by the floating-depot ships.

With the solution at hand, some important inferences can be drawn from the simple example just discussed. For example, if the deployment system were composed of aircraft alone, the full fleet of 50 aircraft would be needed to meet the requirement of C. Alternatively, if only floating-depot ships were used, a total of 40 force units would have to be prestocked to meet the requirement of B. In either case the 10-year deployment system cost would be $400 million. The mixed system, however, with a 10-year system cost of only $334 million, can meet the requirement as well as either pure system and do so at less cost. From another point of view, were B taken to be the worst case because of the magnitude of the tonnage requirement, a comparison of airlift and floating depots would indicate that a floating-depot system containing 40 force units at a cost of $400 million would be needed to meet the requirement, whereas only 40 aircraft at a cost of $320 million would be required. However, the airlift system, although cheaper, would not be able to meet the requirement of C. Alternatively, if C were taken to be the worst case because of its greater distance from A and the shorter deployment period, the cheaper system would be the prestockage of 20 force units in floating depots at a cost of $200 million. Again this system would be incapable of meeting the requirements of 40 force units for B. The advantages to be realized from examining simultaneously the entire set of deployment requirements and all available deployment-system components are evident.

FULL-CONSTRAINT DESCRIPTION USING A SELECTED SUBSET OF CONSTRAINTS AND VARIABLES

The strategic deployment model as structured at present accepts the deployment requirements of five theaters and solves for the least-cost mix of deployment means possessing the capability to meet any one of the individual contingency requirements. Using an alternative formulation of the model, which involves minor modifications, the rapid deployment posture of forces to meet simultaneous requirements of two or more theaters can be optimized.

Inputs to the model are of two general types: The first type treats the characteristics of the available components of the deployment system—costs, capabilities, and constraints on their use. The second type considers the specification of the deployment requirements of the various theaters. The model output information discussed here is also categorized by two types: The first specifies the composition of the least-cost system, and the second gives the level of operation of each component of this system in meeting the requirements.

Deployment requirements and deployment capabilities of each component of the system are dealt with in the model in terms of weight. The unit of measure employed is the kiloton (1000 short tons). In military planning, deployment requirements are usually indicated by specifying the closure dates of individual units. Converted to equivalent tonnages, a typical military planning statement of a deployment requirement is represented by the dotted line in Fig. 2. For

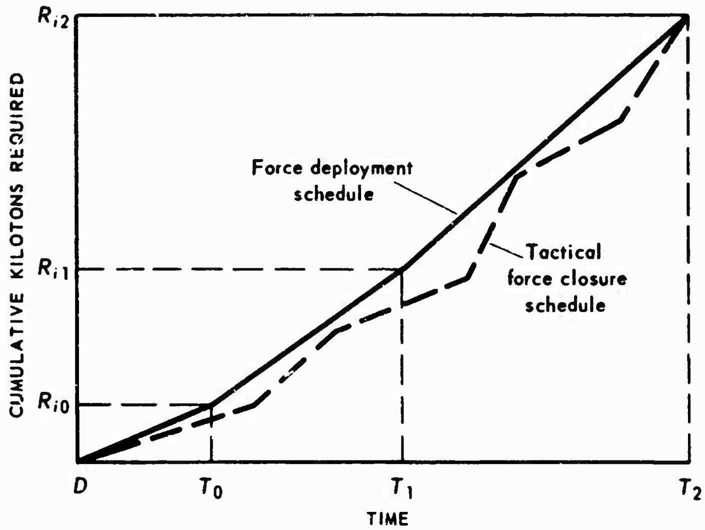

Fig. 2—Typical Tactical and Deployment Schedules

R_{i0}, R_{i1}, R_{i2} are cumulative kilotons required in theater i by times T_0, T_1, T_2.

purposes of the model the deployment requirement is restated by a three-segment linear approximation corresponding to the scheduled times T_0, T_1, T_2. This approximation is represented by the solid line in Fig. 2. Deployment requirements are designated in the model as R_{i0}, R_{i1}, and R_{i2} (for theater i) and represent the cumulative tonnages that must be delivered to the theater by each of the three times scheduled in the deployment period.

A typical utilization of the strategic deployment model has employed approximately 400 constraints and 500 variables. Tonnage deployment requirements by theater and by time period within a theater are specified in the constraints as the objectives to be attained. The variables satisfying the requirements are the overall levels (quantities) of the components of the deployment system and the levels at which the components operate for deployments to each theater.

The next section contains definitions of variables and coefficients used in the sample model presented in this paper. A selected subset of constraints and variables of a complete model is presented in the section, "Objective Function and Constraints for a Strategic Deployment Problem." Included are a linear objective function, a set of linear constraints for one theater for the usual three time periods, and a set of linear constraints for a second theater for only the first two time periods.

Definitions of Variables and Coefficients

Variables and Cost Coefficients.*

W^a, W^b	Number of W-type aircraft with unit costs C^a and C^b respectively
X	Number of X-type aircraft with unit cost D
Y	Number of Y-type ships with unit cost E
Z_s, Z_t	Prepositioned kilotons at sites denoted by s and t with unit costs F_s and F_t respectively
T^W_{ijk}	Kilotons delivered by W-type aircraft from source k
T^X_{ijk}	Kilotons delivered by X-type aircraft from source k
T^Y_{ijr}	Kilotons delivered by Y-type ships, non-mixed-mode,† from initial source r
T^M_{ijr}	Kilotons delivered by Y-type ships and W-type aircraft, mixed-mode,† from initial source r
T^M_{ijs}	Kilotons delivered by Y-type ships and W-type aircraft, mixed-mode,† shuttling from site s

Constraint Coefficients and Limits.*

$B_1, \ldots, B_6$	Limits on system items available
R_{ij}	Cumulative kiloton deployment requirements
PC_{ij}	Kiloton limitation of port throughput capacity
LOC_{ij}	Kiloton limitation on port-to-destination throughput capacity
P^W_{ijk}	W-type aircraft per kiloton from site k
P^X_{ijk}	X-type aircraft per kiloton from site k
P^W_{ijp}	W-type aircraft per kiloton from port p in theater i to forward-area destination
Q^Y_r	Y-type ships per kiloton from initial source r to port p in theater; applies for both non-mixed-mode and mixed-mode activities
K^W, K^X	Ratio of total tonnage lifted to effective tonnage delivered for W-type aircraft and X-type aircraft respectively, where $K^W > 1$, $K^X > 1$
L^W	Number of W-type aircraft required for mixed-mode deployment, in aircraft-days per kiloton
M_{ijr}	Maximum number of days available for initial shipping from r to close, mixed-mode
M_{ijs}	Maximum number of days available for shuttle shipping from site s to close, mixed-mode
M_{ij}	Maximum number of days for all mixed-mode shipping to be effective
N_{ij}	Coefficient of effectiveness of ship-deployed tonnage, non-mixed-mode, where $N_{ij} < 1$

*Subscripts i and j denote the ith theater and the jth time period in the definitions given below. Subscript k denotes the origin of the deployed tonnage, which may be initial site r or prepositioning sites s and t.

†Mixed-mode deployment is defined as the use of Y-type ships to deploy tonnage to the major port in the theater, and subsequent use of W-type aircraft to deploy the tonnage to a forward-area site.

Objective Function and Constraints for a Strategic Deployment Problem*

Objective Function.

minimize

$$C^a W^a + C^b W^b + DX + EY + F_s Z_s + F_t Z_t \qquad \text{(C)}$$

Constraints.

$$\left.\begin{array}{rr}
W^a + W^b \le B_1 & \text{(S1)} \\
X \le B_2 & \text{(S2)} \\
Y \le B_3 & \text{(S3)} \\
Z_s \le B_4 & \text{(S4)} \\
Z_t \le B_5 & \text{(S5)} \\
Z_s + Z_t \le B_6 & \text{(S6)}
\end{array}\right\} \text{Set } S$$

$$\left.\begin{array}{l}
\left.\begin{array}{rr}
W^a + W^b - P^W_{10r}\, T^W_{10r} \ge 0 & \text{(K1)} \\
X - P^X_{10r}\, T^X_{10r} \ge 0 & \text{(K2)} \\
T^W_{10r} + T^X_{10r} \ge R_{10} & \text{(K3)}
\end{array}\right\} \\[2ex]
\left.\begin{array}{rr}
P^W_{10r}\, T^W_{10r} - \sum_k P^W_{11k}\, T^W_{11k} \ge 0 & \text{(K4)} \\
P^X_{10r}\, T^X_{10r} - \sum_k P^X_{11k}\, T^X_{11k} \ge 0 & \text{(K5)} \\
T^W_{10r} + T^X_{10r} + \sum_k T^W_{11k} + \sum_k T^X_{11k} \ge R_{11} & \text{(K6)} \\
Z_s - K^W\, T^W_{11s} - K^X\, T^X_{11s} \ge 0 & \text{(K7)} \\
Z_t - K^W\, T^W_{11t} - K^X\, T^X_{11t} \ge 0 & \text{(K8)}
\end{array}\right\}
\end{array}\right\} \text{Set } R_{10-11}$$

*Note that there are three sources defined by initial location $k = r, k = s, k = t$, and the $\sum_k$ includes these three sources.

$$
\left.
\begin{array}{rr}
W^{a} + W^{b} - \sum_{k} P^{W}_{12k}\, T^{W}_{12k} - P^{W}_{12p}\, T^{M}_{12r} - P^{W}_{12p}\, T^{M}_{12s} \geq 0 & \text{(L1)} \\
X - \sum_{k} P^{X}_{12k}\, T^{X}_{12k} \geq 0 & \text{(L2)} \\
Y - Q^{Y}_{r}\, T^{Y}_{12r} - Q^{Y}_{r}\, K^{W}\, T^{M}_{12r} \geq 0 & \text{(L3)} \\
T^{Y}_{12r} + K^{W}\, T^{M}_{12r} - K^{W}\, T^{M}_{12s} \geq 0 & \text{(L4)} \\
(W^{a} + W^{b})\, M_{12r} - L^{W}\, T^{M}_{12r} \geq 0 & \text{(L5)} \\
(W^{a} + W^{b})\, M_{12s} - L^{W}\, T^{M}_{12s} \geq 0 & \text{(L6)} \\
(W^{a} + W^{b})\, M_{12} - L^{W}\, T^{M}_{12r} - L^{W}\, T^{M}_{12s} \geq 0 & \text{(L7)} \\
T^{Y}_{12r} + K^{W}\, T^{M}_{12r} + K^{W}\, T^{M}_{12s} \leq PC_{12} & \text{(L8)} \\
T^{Y}_{12r} \leq LOC_{12} & \text{(L9)} \\
T^{W}_{10r} + T^{X}_{10r} + \sum_{k} T^{W}_{11k} + \sum_{k} T^{X}_{11k} + \sum_{k} T^{W}_{12k} + \sum_{k} T^{X}_{12k} + N_{12}\, T^{Y}_{12r} + T^{M}_{12r} + T^{M}_{12s} \geq R_{12} & \text{(L10)} \\
Z_{s} - K^{W}\, T^{W}_{11s} - K^{X}\, T^{X}_{11s} - K^{W}\, T^{W}_{12s} - K^{X}\, T^{X}_{12s} - K^{W}\, T^{M}_{12s} \geq 0 & \text{(L11)} \\
Z_{t} - K^{W}\, T^{W}_{11t} - K^{X}\, T^{X}_{11t} - K^{W}\, T^{W}_{12t} - K^{X}\, T^{X}_{12t} \geq 0. & \text{(L12)}
\end{array}
\right\}
\text{Set } R_{12}
$$

$$
\left.
\begin{array}{rr}
W^{a} + W^{b} - P^{W}_{20r}\, T^{W}_{20r} \geq 0 & \text{(M1)} \\
X - P^{X}_{20r}\, T^{X}_{20r} \geq 0 & \text{(M2)} \\
T^{W}_{20r} + T^{X}_{20r} \geq R_{20} & \text{(M3)} \\
P^{W}_{20r}\, T^{W}_{20r} - \sum_{k} P^{W}_{21k}\, T^{W}_{21k} - P^{W}_{21p}\, T^{M}_{21r} \geq 0 & \text{(M4)} \\
P^{X}_{20r}\, T^{X}_{20r} - \sum_{k} P^{X}_{21k}\, T^{X}_{21k} \geq 0 & \text{(M5)} \\
T^{W}_{20r} + T^{X}_{20r} + \sum_{k} T^{W}_{21k} + \sum_{k} T^{X}_{21k} + T^{M}_{21r} \geq R_{21} & \text{(M6)} \\
Y - K^{W}\, Q^{Y}_{r}\, T^{M}_{21r} \geq 0 & \text{(M7)} \\
K^{W}\, T^{M}_{21r} \leq PC_{21} & \text{(M8)} \\
Z_{s} - K^{W}\, T^{W}_{21s} - K^{X}\, T^{X}_{21s} \geq 0 & \text{(M9)} \\
Z_{t} - K^{W}\, T^{W}_{21t} - K^{X}\, T^{X}_{21t} \geq 0 & \text{(M10)}
\end{array}
\right\}
\text{Set } R_{20-21}
$$

The situation represented by the constraints includes as system components two types of W-type aircraft, X-type aircraft, Y-type ships, and prepositioned kilotons at sites s and t. An additional source of kilotons—the basic source r [usually thought of as the continental United States (CONUS)]—is also

available. The three sources are indexed by k, where $k = r$, $k = s$, and $k = t$. In the first period, the interval D to T_0, only the basic source $k = r$ is assumed to be available for delivery, by air only. In the second period, the interval T_0 to T_1, all three sources are available for delivery, by air only. In the third period, the interval T_1 to T_2, the deliveries become more complicated. The previous sources are available for delivery by air. Additional air delivery may be made by W-type aircraft operating in a mixed-mode delivery system with Y-type ships deploying from the basic source $k = r$, and from a prepositioning site $k = s$. Delivery from the basic source may also be made by Y-type ships to the theater moving over a conventional surface line of communication.

The objective function in the problem presented is denoted by C. Constraints are grouped into four sets, termed S, R_{10-11}, R_{12}, and R_{20-21}. In a full version of the model with five theaters and three time periods within each theater, constraint sets R_{22}, R_{30-31}, R_{32}, R_{40-41}, R_{42}, R_{50-51}, and R_{52} would also be included.

The objective function C is the sum of the various subsystem costs, computed by multiplying the unit cost of each component by the activity level of each component. The total cost by component is assumed to be linearly related to the component activity level.

The first set of constraints S appears only once in the model. This set contains the overall constraints on the availability of the components of the deployment system. The constraints $S1, \ldots, S6$ are obvious.

The set denoted by R_{10-11} represents the system utilization in the first theater in the first two time periods. The time periods are denoted by the second element of the operational variable and constraint coefficient subscripts, namely, 0 and 1 for the first and second time periods. Included in this set are constraints on aircraft utilization K1, K2, K4, and K5, and constraints on use of prepositioning K7 and K8. Constraints K3 and K6 express the requirement for delivery of specified cumulative kilotons in the first two periods. In the first time period all deliveries are made by air from the initial source r.* In the second time period, deliveries can be made by air from prepositioning sites s and t as well as from source r.

Constraints K1 and K2 ensure that the available W-type and X-type aircraft equal or exceed the aircraft requirements for carrying kilotons T^W_{10r} and T^X_{10r} from source r. Constraint K3 ensures that the kiloton requirement R_{10} of the first theater in the first period is met.

Constraint K4 requires that the W-type aircraft made available to carry T^W_{10r} in the first period equal or exceed the aircraft which carry the tonnages from all the sources $\Sigma_k T^W_{11k}$ in the second period. This constraint is included since all aircraft are assumed to be CONUS-based initially. Constraint K5 is the same as K4 but for X-type aircraft. Constraint K6 ensures that the cumulative kiloton requirement R_{11} is equaled or exceeded by the tonnages delivered by W-type and X-type aircraft in the first two periods. Constraint K7 requires that the prepositioned kilotons at site s equal or exceed the tonnage lifted by

*The initial source r is usually interpretable as CONUS. It can be structured to represent multiple airfield, ports, and geographical constraints by consideration of times and productivities.

W-type and X-type aircraft. Constraint K8 is the same as K7 but for site t.

The set denoted by R_{12} includes constraints L1 and L2 on aircraft utilization, an overall constraint on ship utilization L3, more specific constraints L4, . . . , L9 on aircraft and ship utilization, a constraint L10 specifying the cumulative tonnages in the first three periods, and prepositioning constraints L11 and L12.

Constraint L1 requires that the W-type aircraft equal or exceed those needed to carry tonnages from all sources $\Sigma_k T^W_{12k}$ plus those needed for mixed-mode delivery of tonnage T^M_{12r} plus those needed for mixed-mode delivery of tonnage T^M_{12s} shuttled from site s. Constraint L2 requires that the X-type aircraft equal or exceed those needed to carry tonnages from all sources $\Sigma_k T^X_{12k}$. Constraint L3 requires that the number of Y-type ships be sufficient to deliver non-mixed-mode tonnage T^Y_{12r} plus total mixed-mode tonnage $K^W T^M_{12r}$. Constraint L4 establishes the constraint that the ships used in shuttling from site s be limited by the initial shipping available. Specifically the tonnage delivered by Y-type ships non-mixed-mode plus the total tonnage delivered by Y-type ships mixed-mode limits the total tonnage delivered by Y-type ships mixed-mode shuttling from prepositioning site s.

The treatment of shipping in the model is not the same as that of air delivery and merits special discussion. The method is simple in concept but complex in structure because of the possible combinations of original ship deliveries and shuttling that may arise. Conceptually a ship is assumed to commence deployment on D-day, and this delivery capability plus all subsequent capabilities are structured into the model. These capabilities are in terms of ship capacity available as a function of time for initial deliveries, both with and without aircraft, and for all shuttling, again with and without aircraft. In the strategic deployment problem illustrated, where only one ship source and one possible shuttle site are assumed, the structure is not complex. In a larger model, however, with five or more initial sources of shipping and up to seven shuttle sites, the pattern is obviously much more complex. More detail of the treatment is presented in the following discussion, particularly in that of constraints L5 to L9.

Constraint L5 states that the maximum number of days available for initial shipping to close in mixed-mode times the available number of W-type aircraft limits the number of aircraft available for handling the mixed-mode tonnage T^M_{12r}. Constraint L6 is similar for mixed-mode tonnage shuttling from site s. Constraint L7 states that the maximum number of days available for all mixed-mode shipping to close times the available number of W-type aircraft must equal or exceed the aircraft requirements for handling tonnages T^M_{12r} plus T^M_{12s}.

Constraint L8 expresses the requirement that kilotons delivered by Y-type ships non-mixed-mode plus kilotons delivered by Y-type ships mixed-mode plus kilotons delivered by Y-type ships shuttling from site s must not exceed the port throughput capacity. Constraint L9 states that kilotons delivered by Y-type ships must not exceed the port-to-destination throughput capacity. Constraint L10 ensures that the cumulative kiloton requirements R_{12} are equaled or exceeded by the kilotons delivered by W-type and X-type aircraft in the first three periods plus the effective kilotons delivered by Y-type ships non-mixed-mode plus the kilotons delivered mixed-mode plus the kilotons delivered by mixed-mode shuttling from prepositioning site s.

Constraint $L11$ requires that the prepositioned kilotons at site s equal or exceed the tonnage lifted by W-type and X-type aircraft in the second and third periods and the tonnage shuttled mixed-mode from site s. Constraint $L12$ is similar, for site t, with no mixed-mode shuttling.

The set of constraints R_{20-21} is presented to show the relation of the system component variables, which are used in all theaters, and the operational variables, which differ from theater to theater. Note that W^a, W^b, X, Z_s, and Z_t appear as in constraints R_{10-11}. Also note that the operational variables are analogous in notation. Additionally R_{20-21} includes in constraints $M4$, $M6$, $M7$, and $M8$ the possibility that a ship deploying to the theater early enough to be used mixed-mode may contribute to the R_{21} requirement. This situation is often encountered in using the model.

COMPUTATION

The model has been run on the IBM 7040 computer at RAC since the first version was developed in early 1964. The LP/40 linear programming computer code,* which is distributed by IBM, has been used in solving the model. Several hundred runs have been made, and over a thousand solutions have been obtained, since 1964. The code can solve linear programming problems with up to 1023 constraints. The number of variables is limited by the tape storage capacity, which for problems with relatively few nonzero constraint coefficients allows for virtually an unlimited number of variables.

The LP/40 code reads in the data of a linear programming problem in the standard format of the SHARE users' organization. The first set of cards contains the row identifications of constraints to be included in the problem. The second set, containing most of the data, includes the nonzero constraint coefficients arranged by column. The third set contains the right-hand side, and the problem can be set up in such a way that solutions may be obtained for more than one right-hand side.

The code permits remarkable flexibility with respect to data input and system operating instructions, which enables the user to carry out diverse investigations quickly and efficiently. In all the applications discussed in this paper, changes in data and operating instructions could be readily accomplished. In no case has it been necessary to modify the LP/40 code.†

USING THE MODEL IN PERFORMING ANALYSES

The model has been presented in a symbolic manner to illustrate its structure. The following discussion outlines investigations performed for the

*A recent reference on LP/40 is "7040/7044 Linear Programming System II (7040-CO-IIX) — User's Manual," H20-046-1, International Business Machines Corporation, 1964.

†It should be noted that the features of computer codes like LP/40 are described in an introductory manner from the potential user's point of view in "An Introduction to Linear Programming," International Business Machines Corporation, E20-8170, 1964.

OASD (SA) at RAC during 1965. The results were used in performing analyses prior to preparation of the FY67 budget with respect to airlift and sealift forces.

Solutions to the various problems that are set up and analyzed do not in themselves furnish adequate information for final decisions other than with respect to order of magnitude. However, comparisons from solution to solution—examining the results of varying such parameters as requirements, costs, and constraint coefficients—provide information very useful in making such decisions.

It is obviously impossible to represent completely the planning problem of strategic deployment in a set of 400—or even many more—linear equations. However, it is possible and reasonable to expect to obtain valuable insights into the problem through the employment of as realistic a system of relations as practicable. In this light, six basic classes of sensitivity analyses have been performed with the model. These classes are defined in terms of the previous terminology as follows.

Class I—Variations in availability of deployment-system components. Various system components were either made available to the model through the use of inequality relations, as in the set S of constraints in the strategic deployment problem illustrated, or forced into the system at some level.

Class II—Variations in time-phased deployment requirements. Each pair of the three deployment period requirements R_{10}, R_{11}, and R_{12} was set in turn to zero to generate for comparison the solution resulting from the third requirement alone.

Class III—Variations in theater requirements. A consistent set of time-phased force requirements was employed while various combinations of theater requirements were investigated. In addition the requirements of single theaters were used individually to determine how the "single-vs-multiple" approach affected the deployment system mix and cost.

Class IV—Parametric cost variations. Costs of one or more items were allowed to range between specified limits of interest. The LP/40 code is capable of giving solutions as costs are varied, computing and printing each new solution as the variables in the system change owing to variations in costs. This is an extremely useful method for assessing the sensitivity of the model solution to costing assumptions.

Class V—Parametric system and requirements variations. These two variations are grouped together because both may be accomplished by using an LP/40 code technique for parametrically varying the right-hand sides of the equations, singly or in groups. Solutions are obtained as the system variables change owing to variation in either requirements or capabilities.

Class VI—Variations in selected matrix elements. Sensitivity analyses of this kind are not well adapted for generalization. In this analysis, certain of the matrix-element coefficients were systematically incremented. The impact of changes in the requirements to support theater lines of communications associated with various deployment modes was examined in this manner.

Computer-generated problem solutions employed in the analyses just enumerated were based, in general, on the available system items shown in Table 1. These data were appropriately varied from problem to problem.

Additional input data included the time-phased deployment requirements of each theater and the deployment capabilities of each type of transport vehicle when deploying from the US or any forward location to any theater in any

TABLE 1

General Input Data for Strategic Deployment Problem

Typical system components available	Quantity available	Remarks
C-130 aircraft	Currently programmed assets	Assumed already programmed; cost includes only 10 years of operation and maintenance
C-141 aircraft	Currently programmed assets	Same as above
C-141 aircraft	Unlimited	Cost of unprogrammed additional aircraft includes procurement cost plus 10 years of operation and maintenance
C-5A aircraft	Unlimited	Cost includes research and development, procurement, and 10 years of operation and maintenance
Victory-ship forward floating depots	Currently programmed assets	Assumed already programmed; cost includes only 10 years of operation and maintenance; delivery capability based on location in Philippines
Victory-ship forward floating depots based in Philippines or Indian Ocean	Unlimited	Cost of unprogrammed additional ships includes procurement of ships, procurement of materiel (including 5 percent annual replacement), and operation and maintenance
Fast-deployment logistic ships CONUS-based East Coast or West Coast	Unlimited	Cost includes research and development, procurement of ships and materiel, and 10 years of operation and maintenance
Fast-deployment logistic ships forward-based Philippines, Okinawa, or Indian Ocean	Unlimited	Same as above
Shore-based prepositioned materiel	Unlimited	Available shore-based sites assumed are Guam, Hawaii, Italy, Okinawa, Philippines, and Turkey; cost of materiel already prepositioned at some sites includes only operation and maintenance; cost of additional materiel includes procurement and operation and maintenance

time period. Vehicle deployment capabilities, as entered in the model, reflected preliminary calculations that accounted for such factors as distance, routes, load and unload times, and vehicle speed.

Output data consisting of optimal system compositions, costs, and deployment profiles were routinely obtained. Postoptimality features of the LP/40 computer code were employed to obtain all information that could be used to assess sensitivity for the types of problems defined by classes I to VI. Hundreds of variations of deployment problems were investigated, and thousands of solutions obtained.

FURTHER USES FOR THE MODEL

The sensitivity data obtained from the model has contributed directly to the formulation of decisions relative to strategic deployment. In particular

the introduction of the C-5A aircraft has been studied. The application of the model in the fashion discussed here has stimulated interest in other applications for a variety of different but related problems where strategic deployment plays a significant role. Several areas of interest which could be explored by the model are (1) planning and budgeting implications of introducing a new type of high-speed forward-based logistic ship for strategic deployment; (2) economic impacts of alteration in world-wide prepositioning structure; and (3) economic impact of changing theater constraints—for instance, port and throughput capacities.

Numerous variations could be made of this basic model of strategic deployment. The model has served as a useful aid to decision making in its present form, and the possibilities for its modification and application to other problems are significant.